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Gradient-based PIV Using Neural Networks

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***Abstract*:** This paper proposes a new gradient-based PIV using an artificial neural network for acquiring the characteristics of a two-dimensional flow field. The neural network can effectively realize an accurate approximation of the vector field by introducing some knowledge on the characteristic property. The neural network is trained by using spatial and temporal image gradients so that the basic equation of the gradient-based method is satisfied. Since the neural network itself learns the stream function, the continuity equation of flow is consequently satisfied in the measured velocity vector field. The new gradient-based PIV can be applied to even partly lacking visualized images.

***Keywords*:** PIV, neural networks, gradient-based method, stream function.

1. Introduction

Particle Imaging Velocimetry (PIV) (Grant, 1997), which is the whole flow field measurement technique using visualized images, has been recognized to be essential and very useful for analyzing complex flow fields. The principle of the conventional measurement methods for PIV is classified into particle tracking and pattern tracking. The pattern tracking methods are particularly applicable to both the solid and liquid tracers for flow visualization and are able to obtain velocity vectors at all the grid points over the flow field. Of them, the gradient-based method (Okuno, 1995), in which an optical flow (a velocity vector profile) is obtained from spatial and temporal image gradients, has often been applied to not only PIV but also Robot Vision just because it has high spatial resolution. In the method, a two-dimensional velocity vector profile is measured by solving a basic equation based on the relationship between two velocity vector components and the image gradients. The method, however, needs another restraint condition because two unknown velocity components are not obtained by solving just one equation. In the conventional methods (Okuno, 1995; Ido et al., 1999), the spatial local and temporal local optimizations are applied to the gradient-based method as the restraint condition. The spatial local optimization, however, does not have high spatial resolution and the temporal local optimization makes temporal resolution worse.

We propose a new method using an artificial neural network for acquiring the characteristics of a two-dimensional flow field, which solves the above problem. The neural network can realize effectively an accurate approximation of the vector field by introducing some knowledge on the characteristic property of fluids. The neural network is trained by using spatial and temporal image gradients so that the basic equation for the gradient-based method is satisfied. Since the neural network itself learns the stream function, the continuity equation of flow is consequently satisfied in the measured velocity vector field. In order to evaluate the effectiveness and accuracy, the method is applied to artificially generated smoke images of a calculated two-dimensional vortex flow. Since the trained neural network is a model of the flow field, the whole field measurement is available even from partly lacking smoke images.

2. Gradient-based PIV Using Neural Networks

2.1 Principle of Gradient-based Method

Figure 1 shows the principle of the gradient-based method (Horn and Schunck, 1981). Supposing that a gray level pattern $f(x, y, t)$ at and around a point (x, y) in a visualized image moves at and around a point $(x + \Delta x, y + \Delta y)$ in a very short time Δt, the following relationship is obtained locally.

$$f(x, y, t) = f(x + \Delta x, y + \Delta y, t + \Delta t) \tag{1}$$

Using Taylor expansion, the right side of the above equation is given by

$$f(x + \Delta x, y + \Delta y, t + \Delta t) = f(x, y, t) + \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial t}\Delta t + O(\Delta x, \Delta y, \Delta t) \tag{2}$$

Since the $O(\Delta x, \Delta y, \Delta t)$ is the term of higher order of small quantities and is negligible, Equations (1) and (2) reach the following relationship.

$$\frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial t}\Delta t = 0 \tag{3}$$

The above equation is divided by Δt.

$$\frac{\partial f}{\partial x}\frac{\Delta x}{\Delta t} + \frac{\partial f}{\partial y}\frac{\Delta y}{\Delta t} + \frac{\partial f}{\partial t} = 0 \tag{4}$$

Considering $\Delta t \to 0$, the basic equation of the gradient-based method is given as follows:

$$\frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} + \frac{\partial f}{\partial t} = 0 \tag{5}$$

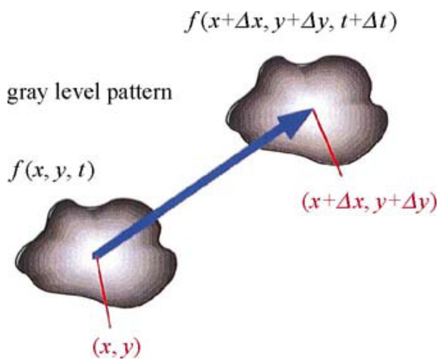


Fig. 1. Principle of gradient-based method.

The above equation is also obtained after assuming non-diffusion in the two-dimensional transportation equation of tracer's concentration. Equation (5) shows the relationship between the two-dimensional velocity vector $(u, v) = (dx/dt, dy/dt)$ and the spatio-temporal derivatives $\partial f/\partial x$, $\partial f/\partial y$ and $\partial f/\partial t$ of gray level pattern obtained from the time series of the images. To measure (u, v), another restraint condition is required because two unknown velocity components are not obtained by solving just one equation. In this paper, an artificial neural network is given as another condition to solve the basic Eq. (5).

2.2 Construction of Neural Network

Supposing that a flow is two-dimensional and incompressible, the stream function Ψ is established since the flow field satisfies the continuity equation. The following relationship is accordingly defined.

$$(u,v)=\left(\frac{\partial\Psi}{\partial y},-\frac{\partial\Psi}{\partial x}\right) \tag{6}$$

The basic Equation (5) is represented using the above Eq. (6) as follows:

$$\frac{\partial f}{\partial x}\frac{\partial\Psi}{\partial y}-\frac{\partial f}{\partial y}\frac{\partial\Psi}{\partial x}+\frac{\partial f}{\partial t}=0 \tag{7}$$

Since the $\partial f/\partial x$, $\partial f/\partial y$ and $\partial f/\partial t$ are approximated as central differences from the images, an unknown quantity is Ψ only and thereby we can solve Eq. (7). The learning problem of the vector field is consequently formulated so that the network can include the relationship expressed by Eq. (7).

The learning rule for correcting synaptic weights in the neural network is shown in Fig. 2. The neural network itself outputs the stream function Ψ corresponding to the inputs (x, y) and then the derivatives of Ψ with respect to x and y are calculated. The network is trained so that the following overall squared error J reaches a minimum.

$$J=\frac{1}{2}\sum\left(\frac{\partial f}{\partial x}\frac{\partial\Psi}{\partial y}-\frac{\partial f}{\partial y}\frac{\partial\Psi}{\partial x}+\frac{\partial f}{\partial t}\right)^2 \tag{8}$$

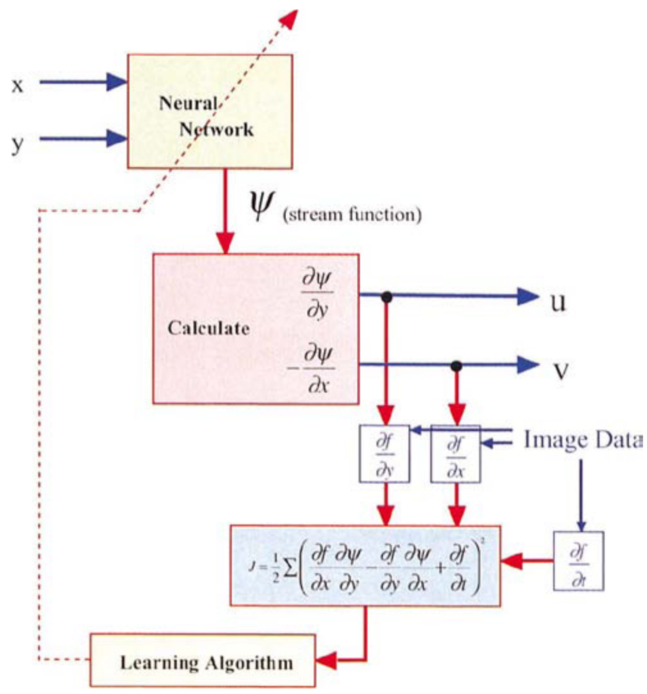


Fig. 2. Schematic diagram for learning rule of the neural network.

After learning, a map of the stream function $\Psi = g(x, y)$ is formed in the neural network. Even if any coordinate values (x, y) of the flow field are inputted to the network, the corresponding stream function Ψ and velocity vector (u, v) are outputted. The above algorithm means that two-dimensional velocity vectors (u, v) are measured so that the basic equation of the gradient-based method is almost satisfied by using the spatial-temporal image data.

To solve the learning problem as mentioned above, we require calculating the derivatives of Ψ with respect to x and y, and the derivative of J with respect to the synaptic weight w_{ij} of the network, which is implicitly included in the stream function, for the steepest decent method. An efficient algorithm for computing those derivatives can be derived by using the adjoint neural network (Kuroe et al., 1998).

3. Artificially Generated Smoke Images of Vortex Flow

3.1 Two-dimensional Vortex Flow

To evaluate the presented gradient-based PIV, a two-dimensional vortex flow given by the following equations is applied (Ido et al., 1999),

$$u = W\sin\left(2\pi k\frac{x}{L}\right)\cos\left(2\pi k\frac{y}{L}\right) \quad v = -W\cos\left(2\pi k\frac{x}{L}\right)\sin\left(2\pi k\frac{y}{L}\right) \tag{9}$$

where W is the amplitude of velocity, L is the side length of the square flow field and k is the wave number. Since smoke images are artificially generated from Eq. (9), the measured results can be evaluated using a correct answer.

Figure 3 shows the calculated velocity vector fields obtained by Eq. (9), where W is 1000 pixel/s, L is 256 pixel and k is 1.0 and 1.5. The coordinates represent the position in the x- and y-directions by pixel.

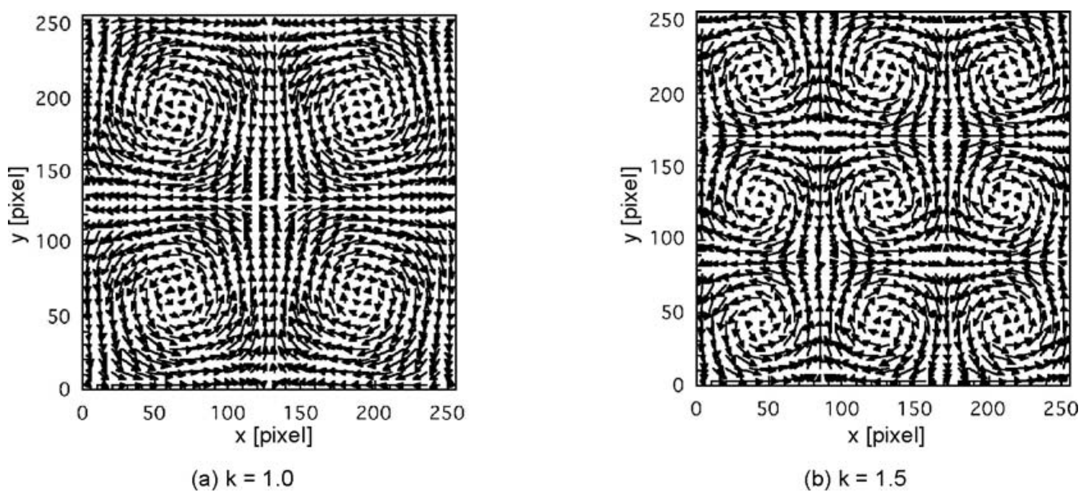


Fig. 3. Calculated velocity vector fields.

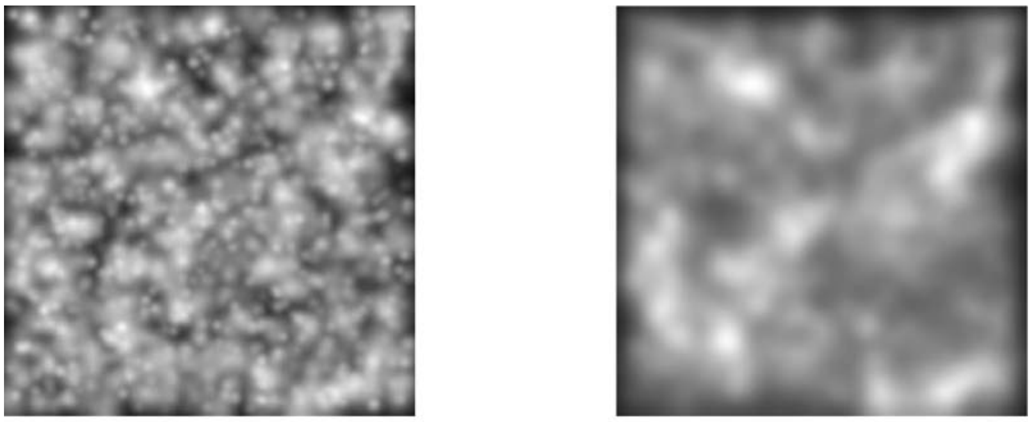
3.2 Generation of Smoke-visualized Images

Based on the puff model, which is a simulation model for transport of atmospheric pollutants, smoke images are generated using the calculated velocity vector fields of Eq. (9). The image size is 256 × 256 pixel and each pixel has 256 gray level. The smoke tracers are approximated by integrating puffs, each concentration of which has an isotropic normal distribution. Each puff independently moves at the speed of its center (u, v) and its standard deviation based on turbulent dispersion $\sigma(t)$, which randomly changes within the range between σ_0 and $3\sigma_0$, is given by

$$\sigma^2(t + \Delta t) = \sigma^2(t) + 2D\Delta t \tag{10}$$

where D is the turbulent diffusion coefficient.

The center coordinates and its concentration of each puff at the time t are determined using random numbers. Figure 4 shows an example of the smoke images. The spatial frequency of the images decreases as parameter σ_0 increases.



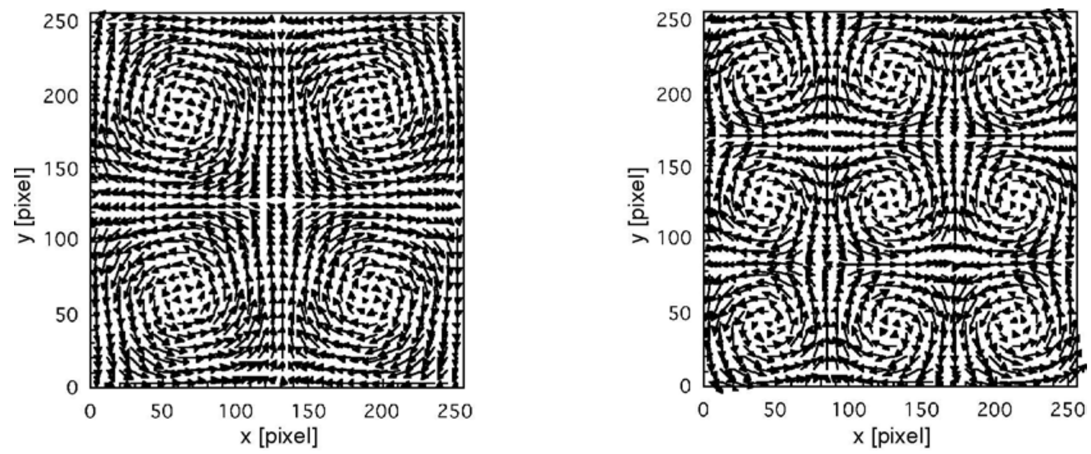
σ_0 = 2 pixel σ_0 = 6 pixel

Fig. 4. Artificially generated smoke images.

4. Application to Vortex Flow Images

4.1 Measurement of Velocity Vector Fields

We apply the proposed gradient-based PIV to the smoke images mentioned above. Since the time interval of the two consecutive images is 1 ms, the moving distance of smoke tracers is under 1 pixel in the *x*- and *y*-directions. Figure 5 shows the measured two-dimensional velocity vector fields. In this case, the neural network comprises three layers of the input, hidden, and output. The number of neuron units in the hidden is 10. Since the images are artificially generated by using Eq. (9), the correct answers of Fig. 5 are those in Fig. 3. All the measured fields agree well with the correct answers.



(a) k = 1.0 (b) k = 1.5

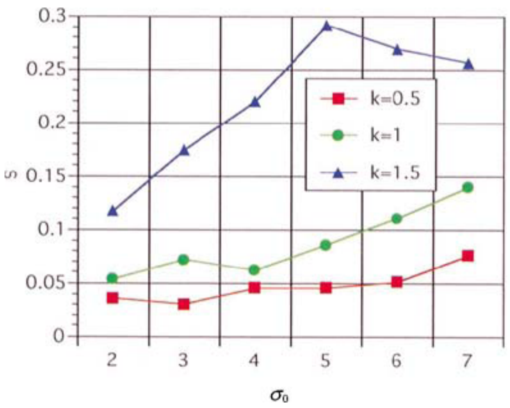
Fig. 5. Measured velocity vector fields.

4.2 Evaluation of the Method

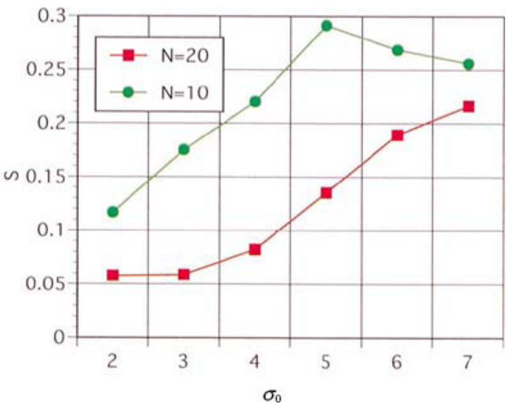
In order to examine the effectiveness and accuracy of the method, the error evaluation index *S* for velocity vectors is defined as

$$S = \frac{\frac{1}{M}\sum_{i=1}^{M}\left|\boldsymbol{v}_j - \boldsymbol{v}_{ci}\right|}{\left|\boldsymbol{v}_{\text{ave}}\right|}, \qquad \boldsymbol{v}_{\text{ave}} = \frac{1}{M}\sum_{i=1}^{M}\boldsymbol{v}_{ci} \tag{11}$$

where $\boldsymbol{v}_i$ is a measured velocity vector, $\boldsymbol{v}_{ci}$ is the corresponding correct velocity vector, $\boldsymbol{v}_{ave}$ is the average velocity and M is the number of all the vectors. Figure 6(a) shows the error evaluation index S versus the parameter σ_0. Although five tests were performed for each condition, almost the same value of the index J was obtained. The index S increases with the increases in the parameter σ_0 and the wave number k, which means that the gradient-based PIV requires the images with high spatial frequency and more units of the hidden layer for a more complex flow. The index S improves when the number of units increases as shown in Fig. 6(b), where N is the number of units in the hidden layer. As a result, the method gives the error evaluation index S of about 0.05 for the images with high spatial frequency.



(a) Effect of wave number k (N =10)



(b) Effect of the number of neuron units N (k =1.5)

Fig. 6. Error evaluation index versus parameter σ_0.

5. Application to Partly Lacking Smoke Images

Seeing the method from a different angle, an approximate model of the flow field is formed from the image data. Accordingly the method does not always need all the image data over the entire measurement area. The neural network can be trained even from deficient image data and it outputs velocity vectors at any points over the field after learning. Figure 7 shows one of partly lacking consecutive images, which are artificially generated by setting a threshold of gray level from the image shown in Fig. 4. Although no image information exists in the black parts of the images, the whole measurement is feasible. The proposed PIV is applied to those deficient images. The velocity vector distribution is obtained as shown in Fig. 8. A good measurement result with the error evaluation index of 0.09 is obtained even from 50% lacking image.

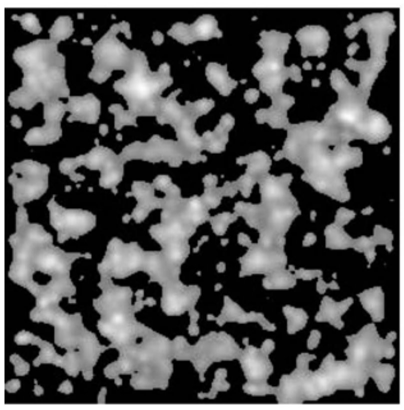


Fig. 7. 50% lacking smoke image.

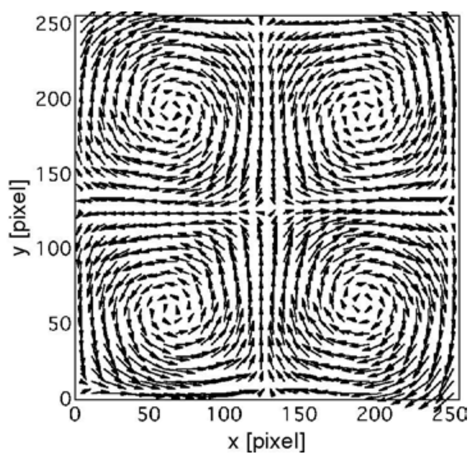


Fig. 8. Velocity vector field measured from Fig. 7 (S = 0.09).

The neural network occasionally falls into a local minimum, which depends on the initial values of synaptic weights, in the case of high percentage lacking images, thereby increasing the error evaluation index. To avoid falling into such a local minimum, the overall squared error J of Eq. (8) must be small enough. The error J can be lowered enough by checking its value at an early stage for learning and then renewing the synaptic weights in case the error J levels off at a higher value.

6. Conclusion

The proposed gradient based PIV gives a correct velocity vector field with high spatial resolution, which automatically satisfies the continuity equation of flow. Besides, there are no conspicuous erroneous vectors in the measured vector fields because the neural network itself learns smoothly the stream function. The method makes the whole field measurement feasible because the neural network is able to estimate the stream function of the entire field even from partly lacking image data. Although a flow is not perfectly two-dimensional in an actual experiment, velocity vector fields with relatively small error will be measured by adding the restraint conditions that a flow is two-dimensional and incompressible.

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